Rectifier circuits & DC power supplies

Goal: Generate the DC voltages – needed for most electronics – starting with the AC power that comes through the power line.

120 V_{RMS} \quad f = 60 \text{ Hz} \quad (T = 16.67 \text{ ms})

V_{ac} = (170\text{V}) \sin \left( \frac{2\pi}{T} t \right)

How to take time-varying voltage with an average value of 0 and turn it into a DC voltage?
transformer : reduces AC amplitude to something safe and manageable. $V_{peak}$ from the transformer will be a few volts bigger than the desired DC voltage.

peak rectifier : breaks up the AC waveform and produces a $V_{DC} \approx V_{peak}$.

regulator : Refines the output of the rectifier. (optional)

Issues:
- Total power
- Efficiency
- Cost
- Load regulation (Does $V_{DC}$ change as the load draws different amounts of current?)
- Line regulation (Does $V_{DC}$ change if the input AC amplitude changes?)
Half-wave rectifier

\[ V_S(t) = V_p \sin \left( \frac{2\pi}{T} t \right) \]

\[ V_p = 3 \text{ V.} \]

Diode is off until \( V_S > 0.7 \text{ V.} \)

Current flows when diode is in forward conduction. The output tracks the input during positive half cycle.

Resistor represents a load.

\( V_R \)

We are trying to deliver DC power to the load.
The diode turns off when $V_S < 0.7 \text{ V}$. It stays off during the negative half cycle of the sinusoid.

$V_S > 0$: $v_R(t) \approx V_p \sin \left( \frac{2\pi}{T} t \right) - 0.7\text{V}$

$V_S < 0$: $v_R(t) = 0$:

$V_o \ (\text{avg}) \approx \frac{V_p}{\pi} - \frac{0.7\text{V}}{2}$

$\neq 0$ !

To get the negative half of the cycle, turn the diode around.
Time delay

Note that since the diode will not turn on until the sinusoid goes above \( \approx 0.7 \text{ V} \), there is time delay before the rectifier “turns on”. It is a simple matter to determine the delay time, using the “on-off” diode model:

\[
0.7V = V_p \sin \left( \frac{2\pi}{T} t' \right)
\]

\[
t' = \frac{T}{2\pi} \arcsin \left( \frac{0.7V}{V_p} \right)
\]

If \( f = 60 \text{ Hz} \) (\( T = 16.67 \text{ ms} \)) and \( V_p = 3 \text{ V} \), \( t' = 0.62 \text{ ms} \).

There is a similar time offset at the other end of the positive half cycle.

The effect of the time offset become negligible if \( V_P \gg 0.7 \text{ V} \).
Peak rectifier

Add a largish capacitor after the diode, in parallel with the load.

Initially, diode is on & cap charges to $V_P - 0.7 \text{ V}$.

While $V_S < v_C$, diode is off! Cap discharges through load.
Diode stay off until $V_S$ comes back around and becomes bigger than $v_C$. Then diode comes on again and re-charges the capacitor.

When $V_S$ falls to less than $v_C$, the diode turn off again, and the cycle continues.
Not a perfect DC voltage at output. There is some variation (ripple) around an average value.

\[
V_o (max) = V_P - 0.7V
\]

\[
V_o (min) = [V_P - 0.7V] \exp\left(-\frac{t_1}{RC}\right)
\]

\[
\approx [V_P - 0.7V] \exp\left(-\frac{T}{RC}\right)
\]

\[
V_{ripple} = V_o (max) - V_o (min)
\]

\[
= [V_P - 0.7V] \left[1 - \exp\left(-\frac{T}{RC}\right)\right]
\]

\[
V_o (avg) \approx V_o (max) - \frac{V_{ripple}}{2}
\]

\[t_1 = \text{time when diode conducts again.}\]

\[t_1 \approx T\]
Example 1

\[ V_S = (15V) \sin \left( \frac{2\pi}{T} t \right) \]

\[ T = 16.67 \text{ ms} \]

Find the average value of \( v_o \) and the ripple voltage. Repeat for \( R = 1000 \Omega \) and 200 \( \Omega \).

\[
V_{\text{ripple}} = [V_p - 0.7V] \left[ 1 - \exp \left( -\frac{T}{RC} \right) \right]
\]

\[
= [15V - 0.7V] \left[ 1 - \exp \left( -\frac{16.67\text{ms}}{(5000\Omega)(100\mu\text{F})} \right) \right]
\]

\[ = 0.47 \text{ V} \]

\[
V_o (\text{avg}) = V_o (\text{max}) - \frac{V_{\text{ripple}}}{2} = 14.3V - \frac{0.47V}{2} = 14.1V
\]

\[
R = 1 \text{ k}\Omega
\]

\[
V_{\text{ripple}} = 2.19 \text{ V}
\]

\[
V_o (\text{avg}) = 13.2 \text{ V}
\]

\[
R = 200 \Omega
\]

\[
V_{\text{ripple}} = 8.09 \text{ V}
\]

\[
V_o (\text{avg}) = 10.2 \text{ V}
\]

Drawing more current causes the ripple to increase and \( V_{\text{DC}} \) to droop. Can fight this with more capacitance.
Example 2

\[ V_s = (25V) \sin \left( \frac{2\pi}{T} t \right) \]

\[ T = 16.67 \text{ ms} \]

Find the capacitance so that the ripple will be no bigger than 1 V.

What is the DC voltage?

\[ V_{\text{ripple}} = [V_P - 0.7V] \left[ 1 - \exp \left( -\frac{T}{RC} \right) \right] \]

\[ C = -\frac{T}{R} \left[ \ln \left( 1 - \frac{V_{\text{ripple}}}{V_P - 0.7V} \right) \right]^{-1} = \frac{16.67 \text{ ms}}{1000 \Omega} \left[ \ln \left( 1 - \frac{1V}{24.3V} \right) \right]^{-1} = 397 \mu F \]

\[ V_o (\text{avg}) = V_o (\text{max}) - \frac{V_{\text{ripple}}}{2} = 24.3V - \frac{1V}{2} = 23.8V \]

What capacitance is needed to limit the ripple to 0.1 V?

\[ C = 4000 \mu F \]
Full-wave rectifier

With a few more diodes, we can rectify the entire sinusoidal input.

The diodes are in a bridge configuration.

During the positive half cycle of the input, diodes 1 and 2 will be forward biased. Current will flow from the positive source through those diodes and the resistor to generate a positive voltage across the resistor.

During the negative half cycle of the input, diodes 3 and 4 will be forward biased. Current will flow from the negative source through those diodes and the resistor to generate a positive voltage across the resistor, again.
Note that there are no two diode drops in the conduction path(s). Also, the frequency is effectively doubled.
Full-wave peak rectifier

Placing a capacitor in parallel with the load, turns the circuit into a full-wave peak rectifier. It behaves essentially the same as the half-wave peak rectifier except with twice the frequency (half the period).

\[ V_S(t) = V_p \sin \left( \frac{2\pi}{T} t \right) \]

\[ V_p = 8 \text{ V.} \]

The ripple voltage is calculated in exactly the same way, except that the period is cut in half (frequency doubled).

\[ V_{\text{ripple}} = [V_P - 1.4V] \left[ 1 - \exp \left( -\frac{T}{2RC} \right) \right] \]

Same as doubling capacitance!
Example 3

You want to use a wall transformer that produces $10\text{ V}_{\text{RMS}}$ at the secondary to generate a DC voltage. The desired voltage DC should be greater than 12 V and it should be able to supply at least 50 mA while keeping the voltage ripple to less than 5%. Design the rectifier to meet these goals. (Note: $f = 60 \text{ Hz}$.)

$10 \text{ V}_{\text{RMS}} \rightarrow 14.1 \text{ V}$ amplitude

effective $R_L \approx \frac{V_o}{I_o} = 12.0 \text{ V} / (50 \text{ mA}) = 240 \Omega$

Note: This would be the minimum value of effective resistance. If we choose $C$ to meet the ripple requirement, then we will still be safe if we use a slightly higher $V_o$.

Two options: half-wave or full-wave rectifier. Try both.

Half-wave:

$V_o(\text{max}) = V_p - 0.7 \text{ V} = 13.4 \text{ V} \rightarrow V_{\text{ripple}} \leq 0.67 \text{ V}$.

$$C = -\frac{T}{R} \left[ \ln \left( 1 - \frac{V_{\text{ripple}}}{V_P - 0.7V} \right) \right]^{-1} = 1350 \mu\text{F}$$

$V_o(\text{avg}) = V_o(\text{max}) - V_{\text{ripple}} / 2 = 13.06 \text{ V}.$
Full-wave:

\[ V_o(\text{max}) = V_p - 2(0.7 \text{ V}) = 12.74 \text{ V} \rightarrow V_{\text{ripple}} \leq 0.64 \text{ V}. \]

\[ C = -\frac{T}{2R} \left[ \ln \left( 1 - \frac{V_{\text{ripple}}}{V_o(\text{max})} \right) \right]^{-1} = 673 \mu \text{F} \]

\[ V_o(\text{avg}) = V_o(\text{max}) - \frac{V_{\text{ripple}}}{2} = 12.42 \text{ V}. \]

Either approach will work and meet the requirements. The full-wave version uses extra diodes, but only half the capacitance. Since diodes are nearly free (pennies per piece), but big capacitors are relatively expensive, the full-wave circuit will actually cost less than the half-wave.

This is why full-wave rectifiers are used more commonly than half-wave rectifiers.

Component manufactures supply full-wave bridge rectifiers packaged as single unit with the transformer sinusoid as input the rectified waveform as the output.