Charge

Charge is a fundamental property of the particles that make up all matter.

• protons – one positive unit of charge, $+q$
• electrons – one negative unit of charge, $-q$
• neutrons – no charge (so are not very interesting to EE/CprEs)

Charge is important because there are strong forces between charges. From the electromagnetics part of your basic physics class, you might recall that if two charges, $Q_1$ and $Q_2$, are separated by a distance $r$, the force between them is given by Coulomb’s law:

$$F = \frac{Q_1 Q_2}{4\pi \epsilon_0 r^2}$$

If the charges have the same sign, the force is repulsive (like charges are pushed apart) and if of opposite signs, the force is attractive (opposites attract).
For reasons of convenience, the force fields that push and pull on charges are described as “electric fields”. The value of an electric field is simply the force divided by an amount of charge.

\[ \mathcal{E} = \frac{F}{Q} \]

If we have charge of amount \( Q_1 \), fixed in space, it will have an electric field around it

\[ \mathcal{E} = \frac{Q_1}{4\pi\varepsilon r^2} \]

If we bring a second mobile charge near the first, the second charge will have a force pushing it away from or pulling it towards the first charge:

\[ F = Q_2\mathcal{E} = \frac{Q_1 Q_2}{4\pi\varepsilon r^2} \]
Current

So, by creating electric fields, we have a mechanism by which we can move charges around. Because the charged particles are so light and the forces on them are so strong, the charges are easily moved.

The movement of charge is called **current**. Conceptualizing current is a simple thing: Stand as some point and count how much charge passes by in a given amount of time. (Think of standing along a roadside and counting the cars that pass.) It’s important to remember that it is the quantity of charge passing and not the number of particles, although the two are clearly related.

\[ I = \frac{Q}{\Delta t} \]

Another way to view current is to consider a large quantity or container of charge. If the amount of charge in the container is changing (increasing or decreasing) with time, there must a current flowing in or flowing out. (Think of watching a parking and noting that the number of filled spots changes with time.) The time-rate-of-change of the amount of stored charged must be equal to the current in or out.

\[ I = \frac{dQ}{dt} \]
So we already have the broad outlines of what happens in circuits: We create forces (electric fields), which in turn cause charges to move (currents). There’s really not much more to it than that. Of course, the details are more involved.

In circuits, we generally don’t deal directly with electric fields very often. (From time to time, they pop up in our descriptions.) Instead, we describe the effects of fields in terms of the energy changes involved when a charged particle moves in that field.

Recall again from basic physics, that a force field can be defined in terms of a potential energy.

\[ F = \frac{dE}{dx} \]

This relation says that a force can be described in terms of an energy function which varies from one point to another.
Voltage

With electric forces, we scale the force by the charge, so that we can use the more useful quantity of electric field.

\[ \mathcal{E} = \frac{F}{Q} = \frac{d(E/Q)}{dx} \]

We see that there is a quantity \( E/Q \) which scales the electric energy of the charge by that amount of charge. We call this quantity the *electrostatic potential*, or, more commonly, the *voltage*. Usually denoted by the variable \( V \). (In other places, you may see it denoted by \( \phi \).)

\[ \mathcal{E} = \frac{dV}{dx} \]

(Sometimes voltage may be referred to as EMF or electromotive force. This seems misleading, since voltage is not a force. We won’t use this terminology in 201.)
We can try to view things in terms of cause and effect.

If there is a difference in voltage (energy) between two points, there is an electric field (force) between those points. The field can cause charges to move (current).

\[ \Delta V \rightarrow \mathcal{E} \rightarrow I \]

In circuit analysis, we usually skip the electric field part of this sequence, and say that differences in voltage lead to currents — and vice versa. (We will electric fields to the domain of electromagnetics — mostly.)

\[ \Delta V \leftrightarrow I \]

In EE 201, all of our efforts will focus on how voltage relates to current in various electrical components and the circuits made up of those components.
Units

The unit of charge is the coulomb (C).

Current is the amount charge that passes in a given time, so the units are coulombs / second (C/s). Current has its own units: amperes (A). 1 ampere = 1 C/s = 1 A. (Amperes is sometimes shortened to “amps”.)

Oddly enough, even though charge would seem to be the more fundamental quantity, with current derived from that, the units are defined in the opposite way. One coulomb is defined as the amount of charge that moves in one second when a current of one amp is flowing.

Voltage (or electrostatic potential) is the amount of energy per unit charge, so units are joules / coulomb. Voltage also has its own units: volts (V). 1 volt = 1 J/Q = 1 V.

The use of “volts” for both the quantity and the unit can be confusing, particularly when we start writing equations. It is important to keep the distinction clear. (That is why it is sometimes less confusing to use “potential” in place of “voltage”.)
A proton carries a positive charge of $1.6 \times 10^{-19}$ C – an extremely small number. Viewing this another way, a current of protons that equals 1 A means that there are $6.25 \times 10^{18}$ protons moving past a point in the circuit in 1 second.

An electron carries exactly the opposite charge from that of the proton, $-1.6 \times 10^{-19}$ C. So a current consisting of electrons requires similarly large number of particles.

Why is the electron negative and the proton positive?

Interesting observation: A current consisting of negative charge flowing to the left is identical to a current of positive charge flowing to the right. You can’t tell the difference.

**Important:** For EE 201, we will always assume that the current consists of positive charge carriers. We will pretend as if negative charge never flows in a current.

When we get to transistors (EE 230 and onward), we will need to worry about positive and negative charge.
Typical values

In studying circuits, we will always be discussing voltage and currents, and naturally, we will use the units of volts and amps. However, calculations can be made more “user-friendly” with prefixes on the units.

0.001 volts = 1 millivolt (1 mV), $10^{-6}$ volts = 1 microvolt (1 µV).

1000 volts = 1 kilovolt (1 kV), $10^6$ volts = 1 megavolt (1 MV).

Be careful with the distinction between mV and MV.

0.001 amps = 1 milliamp (1 mA), $10^{-6}$ amps = 1 microamp (1 µA), $10^{-9}$ amps = 1 nA.

1000 amps = 1 kiloamp (1 kA), $10^6$ amps = 1 mega-amp (1 MA).

Electronics: V or mV. Sometimes µV.

mA or µA. Occasionally A. Occasionally nA.

Power Systems: V or kV. Sometimes MV.

A or kA. Occasionally MA.
Some numbers

Q: How many electrons make up a collection of charge totaling –5 nC?
A: 5 nC = 5x10^{-9} C. At 1.6x10^{-19} C per electron, that amount of charge would total 5x10^{-9}/1.6x10^{-19} = 3.125x10^{10} electrons.

Q: A capacitor has 50 quadrillion electrons on one plate. How much charge is that?
A: 50 quadrillion electrons corresponds to (50x10^{15})(–1.6x10^{-19} C) = –8 mC.

Q: 10^{20} electrons are flowing past a particular point in a wire each minute. What is the current?
A: \( i = \frac{\Delta Q}{\Delta t} = (10^{20})(1.6x10^{-19} C)/(60 \text{ s}) = 0.267 \text{ A} = 267 \text{ mA}. \)

Q: A current of 0.5 A is flowing through a 60-W light bulb. How many electrons flow through the bulb each minute?
A: \( i = \frac{\Delta Q}{\Delta t}. \Delta Q = i\Delta t = (0.5 \text{ C}/\text{s})(60 \text{ s}) = 30 \text{ C}. \) 30 C represents (30 C)/(1.6x10^{-19} \text{ C}) = 1.875x10^{20} electrons.