Linear oscillators

We can turn the idea of trying to make amplifiers stable on its head by taking a nominally stable amplifier and adding a feedback circuit that will cause the closed-loop system to become unstable. We use positive feedback – a sample of the output feed back and added to the input.

![Feedback network diagram](image)

The feedback network has a very specific frequency dependence, $\beta \rightarrow \beta(s)$. With positive feedback, the closed-loop transfer function is

$$G(s) = \frac{A}{1 - A\beta(s)}$$

If the frequency-dependence of the feedback circuit relies on an LC resonance, it is usually referred to as a “tank circuit”.
To create the instability, the denominator of the closed-loop transfer function must be zero.

$$1 - A\beta(s) = 0$$

Which is to say that the loop gain, $A\beta(s)$, must be equal to 1.

$$A\beta(s) = 1$$

$$A\beta(j\omega) = 1e^{j0^\circ}$$ This is known as the Barkhausen criterion.

Another way of describing what is happening is that the poles of the transfer function must occur in the right-half plane. Ideally, the poles would be right on the imaginary axis, meaning that the denominator has zeros at $s = \pm j\omega_0$. Thus $1 - A\beta(s)$ should have a factor of the form $s^2 + \omega_0^2$.

In practice, it is very difficult to design a circuit that has poles exactly on the imaginary axis. Generally, the circuit is designed to have poles that are slightly in the right-half plane. Then the oscillations will grow exponentially with time and some sort of amplitude control will be needed to keep the oscillations close to sinusoidal.
The Wien-bridge circuit.

A simple and popular oscillator circuit is based on the Wein bridge. The circuit is essentially a non-inverting amp with a frequency-dependent voltage divider connecting the output back to the non-inverting input.

While not a requirement, the circuit is usually designed with \( R_S = R_P \) and \( C_S = C_P \).

The loop gain is easily calculated.

\[
\begin{align*}
\nu_o &= \left(1 + \frac{R_2}{R_1}\right) \nu_+ \\
\nu_+ &= \frac{Z_p}{Z_p + Z_s} \nu_o = \frac{R_p}{1+sR_pC_p} \nu_o = \frac{1}{1 + \frac{R_S}{R_P} + \frac{C_P}{C_S} + sR_S C_P + \frac{1}{sR_P C_S}} \nu_o
\end{align*}
\]

Note: Positive feedback!
If the resistors are equal \((R_P = R_S = R)\) and the capacitors are equal \((C_P = C_S = C)\), the loop gain is

\[
A\beta(s) = \frac{1 + \frac{R_2}{R_1}}{3 + sRC + \frac{1}{sRC}}
\]

substituting \(s = j\omega\),

\[
A\beta(j\omega) = \frac{1 + \frac{R_2}{R_1}}{3 + j(\omega RC - \frac{1}{\omega RC})}
\]

\[
\theta_{A\beta} = -\arctan\left(\frac{\omega RC - \frac{1}{\omega RC}}{3}\right)
\]

Obviously, to have the phase angle be zero, \(\omega_0 RC = 1/(\omega_0 RC)\), which occurs when \(\omega_0 = 1/(RC)\). The oscillation frequency is determined by the \(RC\) product.

Also, to make the magnitude be 1 at the oscillation frequency, we must have the non-inverting gain be \(1 + R_2/R_1 = 3\) or \(R_2/R_1 = 2\).
A PSPICE example.

Use the 741 op amp model in PSPICE. $R_P = R_S = 10\ \text{k}\Omega$. $C_P = C_S = 16\ \text{nF}$. The expected oscillation frequency is 1 kHz.

SPICE needs some kind of transient to “kick-start” the oscillation. Use a single short pulse at the input to get things started. Use transient analysis to see voltage as a function of time.
With $R_2 = 35 \text{ k}\Omega$ and $R_1 = 25 \text{ k}\Omega$, there is not enough gain to start the oscillation. $(1 + \frac{R_2}{R_1}) = 2.4.$
Increasing the gain to 2.86 ($R_2 = 39 \, \text{k}\Omega$ and $R_1 = 21 \, \text{k}\Omega$), gives a few more wiggles, but the gain is still too small to sustain the oscillations.
With $R_2 = 40 \, k\Omega$ and $R_1 = 20 \, k\Omega$, the gain is exactly 3 and the circuit oscillates with a clean sine wave.
With $R_2 = 41\, k\Omega$ and $R_1 = 19\, k\Omega$, the gain is 3.16 and the oscillations grow with time until clipped by the power supply limits.

The poles of the transfer function are now in the right-half plane, meaning that this is an unstable exponential growth in the response.
Increasing the gain even more ($R_2 = 35 \, \text{k}\Omega$ and $R_1 = 15 \, \text{k}\Omega$ giving a gain of 3.33) causes the oscillations to grow – and clip – even faster.

The poles are even farther into the right-half plane, making the exponential growth that much stronger.
phase-shift oscillator

Uses an inverting amp for the gain, so there is $180^\circ$ phase shift with that. The tank circuit must provide another $180^\circ$ of phase shift to get back to $0^\circ$ ($360^\circ$) to meet the Barkhausen criterion. This requires a 3-pole circuit, since a 2-pole circuit will get to $180^\circ$ only at $f \rightarrow \infty$. 

![Phase-Shift Oscillator Diagram](image-url)
Calculate the loop gain. (Break the loop somewhere…)

\[ sC(v_1 - v_x) = \frac{v_x}{R} + sC(v_x - v_y) \]

\[ sC(v_x - v_y) = \frac{v_y}{R} + sC(v_y) \]

\[ sC(v_y) = -\frac{v_o}{R_F} \]

\[ L(s) = \frac{V_o}{V_1} = -\frac{\left(\frac{R_F}{R}\right) \left(s^2R^2C^2\right)}{4 + \frac{1}{sRC} + 3sRC} \]
Switching to AC analysis \((s = j\omega)\)

\[
L(j\omega) = \left(\frac{R_F}{R}\right) \frac{\omega^2 R^2 C^2}{4 + j(3\omega RC - \frac{1}{\omega RC})}
\]

Phase will be zero when

\[
3\omega_o RC - \frac{1}{\omega_o RC} = 0
\]

\[
\omega_o = \frac{1}{\sqrt{3}RC}
\]

Required gain at the oscillation frequency is

\[
|L| = \frac{R_F}{12R} = 1
\]
quadrature oscillator

This is an interesting one. It uses an straight integrator circuit followed by a non-inverting amp with a single-pole tank circuit. The output of each op-amp will oscillate sinusoidally, and the two sinusoids are 90° out of phase (i.e. in quadrature). This can be useful in some applications.
Calculate the loop gain. (Break the loop somewhere…)

\[ V_2 = V_{o2} \]

From the integrator: \[ V_{o1} = -\frac{1}{sR_1C} V_1 \]

At node x: \[ \frac{V_{o1} - V_x}{R_2} + \frac{V_{o2} - V_x}{R_f} = sCV_x \]

From the inverting amp: \[ V_{o2} = \left(1 + \frac{R_4}{R_3}\right) V_x \]
Putting it all together, we can find the loop gain:

\[ L(s) = \frac{V_2}{V_1} = -\frac{1 + \frac{R_4}{R_3}}{s \left[ 1 - \frac{R_2 R_4}{R_2 R_f} \right] R_1 C + s^2 R_1 R_2 C^2} \]

Switching to AC analysis \((s = j\omega)\)

\[ L(j\omega) = -\frac{1 + \frac{R_4}{R_3}}{j\omega \left[ 1 - \frac{R_2 R_4}{R_2 R_f} \right] R_1 C - \omega^2 R_1 R_2 C^2} \]

The phase will be zero (Barkhausen criterion) when

\[ \frac{R_2 R_4}{R_2 R_f} = 1 \]

When the phase is zero, the magnitude is

\[ |L| = \frac{1 + \frac{R_4}{R_3}}{\omega^2 R_1 R_2 C^2} \]
The magnitude needs to be 1 (or bigger) at the oscillation frequency. There are many ways to choose component values to meet the two conditions. One common and simple combination is:

\[ R_2 = R_3 = R_4 = R_f = 2R_1 \]

In that case, the loop gain reduces to:

\[ L(s) = -\frac{1}{s^2R_1^2C^2} \]
\[ L(j\omega) = \frac{1}{\omega^2R_1^2C^2} \]

The oscillation will occur when

\[ |L| = \frac{1}{\omega_0^2R_1^2C^2} = 1 \]
\[ \omega_0 = \frac{1}{R_1C} \]

\( R_f \) is typically a potentiometer, which can be adjusted to bring the circuit to the oscillation condition. Adjusting so that \( R_f = 2R_1 \) brings the circuit to the onset of oscillation. Making \( R_f \) smaller guarantees oscillation, at the cost of linearity of the sinusoid.

Because \( v_{o1} \) is the integral of \( v_{o2} \), it will be shifted in phase by 90°. Also, the filtering action of the integrator circuit tends to make \( v_{o1} \) more linear (i.e. have less distortion) than \( v_{o2} \).