Voltage/current dividers

Voltage and current dividers are easy to understand and use. They are so easy that it may seem not worth the bother of learning them as a separate techniques. But the divider methods, when combined with the equivalent resistances, may be the most used technique in electronics. Knowing how to use dividers will allow us to quickly recognize what is happening in a circuit and determine important voltages and currents.

An engineer could certainly analyze and design circuits without having voltage and dividers in their “tool bag”, but they would be wasting lots of time writing unnecessary KVL and KCL equations.
Voltage divider

Consider a portion of circuit that has several resistors in series, like the circuit at right. Suppose we want to find the voltage across $R_2$.

We could start by finding the current, which would be equal to the source voltage divided by the equivalent resistance of the string.

$$i_S = \frac{V_S}{R_{eq}}$$

For the series string, $R_{eq} = R_1 + R_2 + R_3$.

Then the voltage across $R_2$ is just

$$v_{R2} = R_2 \cdot i_S = \frac{R_2}{R_{eq}} \cdot V_S = \frac{R_2}{R_1 + R_2 + R_3} \cdot V_S$$

The total voltage is divided among the resistor in the string. The fraction of the voltage across $R_2$ is given by a simple resistor ratio.
The other resistor voltages are calculated just as easily.

\[
\begin{align*}
v_{R1} &= \frac{4.7 \text{ kΩ}}{4.7 \text{ kΩ} + 15 \text{ kΩ} + 10 \text{ kΩ}} (15 \text{ V}) = 2.37 \text{ V} \\
v_{R2} &= \frac{15 \text{ kΩ}}{4.7 \text{ kΩ} + 15 \text{ kΩ} + 10 \text{ kΩ}} (15 \text{ V}) = 7.58 \text{ V} \\
v_{R3} &= \frac{10 \text{ kΩ}}{4.7 \text{ kΩ} + 15 \text{ kΩ} + 10 \text{ kΩ}} (15 \text{ V}) = 5.05 \text{ V}
\end{align*}
\]

The three divided voltages sum up to \( V_S \), as KVL insists. If we insert some numbers: \( V_S = 15 \text{ V}, \ R_1 = 4.7 \text{ kΩ}, \ R_2 = 15 \text{ kΩ}, \) and \( R_3 = 10 \text{ kΩ} \).

It's that easy.
Current divider

Same idea, but with parallel resistors dividing a current. Suppose we want to know the current through $R_2$.

We could start by finding the voltage, which would be equal to the source current multiplied by the equivalent resistance of the parallel resistors.

$$v_S = I_S \cdot R_{eq}$$

For the parallel combination, $R_{eq} = \frac{1}{\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}}$

Then the current through $R_2$ is

$$i_{R2} = \frac{v_S}{R_2} = \frac{R_{eq}}{R_2} \cdot I_S = \frac{1}{\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}} \cdot I_S$$

As in the case of the voltage divider, the fraction of the current through one resistor is determined by a simple ratio based on resistor values. But in the current case, resistor inverses are used.
The other resistor currents are calculated just as easily.

\[ i_{R1} = \frac{1}{R_1} \cdot I_S \]

\[ i_{R2} = \frac{1}{R_2} \cdot I_S \]

\[ i_{R3} = \frac{1}{R_3} \cdot I_S \]

The three divided current sum up to \( I_S \), as KCL insists. If we insert some numbers: \( I_S = 15 \text{ mA} \), \( R_1 = 2.2 \text{ k}\Omega \), \( R_2 = 3.3 \text{ k}\Omega \), and \( R_3 = 6.8 \text{ k}\Omega \).

\[ i_{R1} = \frac{1}{2.2 \text{ k}\Omega} + \frac{1}{3.3 \text{ k}\Omega} + \frac{1}{6.8 \text{ k}\Omega} \cdot (15 \text{ mA}) = 7.54 \text{ mA} \]

\[ i_{R2} = \frac{1}{2.2 \text{ k}\Omega} + \frac{1}{3.3 \text{ k}\Omega} + \frac{1}{6.8 \text{ k}\Omega} \cdot (15 \text{ mA}) = 5.02 \text{ mA} \]

\[ i_{R3} = \frac{1}{2.2 \text{ k}\Omega} + \frac{1}{3.3 \text{ k}\Omega} + \frac{1}{6.8 \text{ k}\Omega} \cdot (15 \text{ mA}) = 2.44 \text{ mA} \]
In many instances, the combination of dividers with equivalent resistances provides for fast calculation of voltages and currents.

**Example 1**

In the circuit at right, find \( v_{R4} \).

Since \( R_3 \) and \( R_4 \) are in parallel, they have the same voltage and we can use the parallel equivalent. We can also find the parallel equivalent of \( R_1 \) and \( R_2 \).

\[
R_{34} = \frac{R_3 R_4}{R_3 + R_4} = 333 \Omega
\]

\[
R_{12} = \frac{R_1 \cdot R_2}{R_1 + R_2} = 320 \Omega
\]

Then use a voltage divider on the simplified circuit.

\[
v_{R4} = \frac{R_{34}}{R_{12} + R_{34} + R_5} \cdot V_S = 4.065 \text{ V}
\]
Example 2

In the circuit at right, find $i_{R2}$.

Since $R_1$ and $R_2$ are in series, they have the same current, and we can use the series equivalent. We can also find the equivalent resistance of the branch with $R_4 - R_5 - R_6$.

\[
R_{12} = R_1 + R_2 = 51 \, \Omega
\]

\[
R_{456} = \frac{R_4 \cdot R_5}{R_4 + R_5} + R_6 = 153 \, \Omega
\]

Then use a current divider on the simplified circuit.

\[
i_{R2} = \frac{1}{\frac{1}{R_{12}} + \frac{1}{R_3} + \frac{1}{R_{345}}} \cdot I_S = 1.19 \, \text{A}
\]
Example 3

Find $v_{R2}$ and $v_{R4}$ in the circuit.

We solve this using the voltage divider calculation twice in succession. First we find $v_{R2}$ using a voltage divider formed by $R_1$ and the equivalent resistance of $R_2$ in parallel with the series combination of $R_3$ and $R_4$.

$$R_{234} = R_2 \parallel (R_3 + R_4)$$

$$= (3.3 \, \text{k}\Omega) \parallel (6.6 \, \text{k}\Omega)$$

$$= 2.2 \, \text{k}\Omega$$

$$v_{R2} = \frac{R_{234}}{R_{234} + R_1} \cdot V_S = 6 \, \text{V}$$

Then the voltage $v_{R2}$ is divided between $R_3$ and $R_4$.

$$v_{R4} = \frac{R_4}{R_3 + R_4} \cdot v_{R2} = 3 \, \text{V}$$
Example 4

Find $i_{R3}$ in the circuit.

Similar to example 3, we can cascade dividers to find the current in two steps.

$I_s$ splits between $R_1$ and the branch with $R_2 - R_3 - R_4$. To find the current through $R_2$, we use the equivalent resistance of that branch, which forms a current divider with $R_1$. Then $i_{R2}$ is divided between $R_3$ and $R_4$.

\[
R_{234} = R_2 + R_3 || R_4 = 3.3 \, \text{k}\Omega
\]

\[
i_{R2} = \frac{1}{\frac{1}{R_{234}} + \frac{1}{R_1}} \cdot I_s = 0.4 \, \text{A}
\]

\[
i_{R3} = \frac{1}{\frac{1}{R_3} + \frac{1}{R_4}} \cdot I_{R2} = 0.2 \, \text{A}
\]
Example 5

Find the voltage $v_x$ indicated in the circuit at right.

By KVL, $v_x = v_{R2} - v_{R4}$.

We see that $R_1$ and $R_2$ form a voltage divider splitting $V_S$. The same for $R_3$ and $R_4$.

Using voltage dividers.

$$v_{R2} = \frac{R_2}{R_1 + R_2} \cdot V_S = 66 \text{ V}$$

$$v_{R4} = \frac{R_4}{R_3 + R_4} \cdot V_S = 10 \text{ V}$$

Then

$$v_x = 66 \text{ V} - 10 \text{ V} = 56 \text{ V}.$$
Example 6

Find the voltage $v_x$ indicated in the circuit at right.

As with the previous example, KVL tells us that $v_x = v_{R2} - v_{R4}$.

We see that the series combination $R_1 + R_2$ forms a current divider with the series combination $R_3 + R_4$, splitting $I_S$ between the two branches.

Using current dividers.

$$i_{R1} = \frac{1}{\frac{1}{R_1 + R_2} + \frac{1}{R_3 + R_4}} \cdot I_S = 10 \text{ mA}$$

$$i_{R3} = \frac{1}{\frac{1}{R_3 + R_4} + \frac{1}{R_1 + R_2}} \cdot I_S = 50 \text{ mA}$$

Then

$$v_x = i_{R1}R_2 - i_{R3}R_4 = 28 \text{ V}.$$
Example 7

In the circuit at right, the switch can be opened or closed to control the voltage across $R_3$. When the switch is closed ($R_1$ shorted out), $v_{R3}$ is twice as big as the case when the switch is open ($R_1$ not shorted.) How is $R_1$ related to $R_2 + R_3$?

There are several approaches to answering this question, but using voltage dividers is a convenient method. With the switched closed, $R_1$ is shorted out and

$$v_{R3} = \frac{R_3}{R_2 + R_3} \cdot V_S$$

With the switch open, $R_1$ is part of the divider:

$$v'_{R3} = \frac{R_3}{R_1 + R_2 + R_3} \cdot V_S$$

$$v_{R3} = 2v'_{R3}$$

$$\frac{R_3}{R_2 + R_3} \cdot V_S = \frac{2R_3}{R_1 + R_2 + R_3} \cdot V_S$$

$$R_1 + R_2 + R_3 = 2 (R_2 + R_3)$$

$$R_1 = R_2 + R_3$$
Example 8

In the circuit at right, the two switches can be opened or closed to control the current through $R_3$. Calculate the current through $R_3$ for all combinations of the switches being open closed.

$S_1$ open and $S_2$ open: $i_{R3} = I_S = 100$ mA

$S_1$ open and $S_2$ closed: $i_{R3} = \frac{1}{\frac{1}{R_3} + \frac{1}{R_3}} \cdot I_S = 50$ mA

$S_1$ closed and $S_2$ open: $i_{R3} = \frac{1}{\frac{1}{R_1} + \frac{1}{R_3}} \cdot I_S = 33.3$ mA

$S_1$ closed and $S_2$ closed: $i_{R3} = \frac{1}{\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}} \cdot I_S = 25$ mA